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Sobolev embeddings in Musielak-Orlicz spaces

An embedding theorem for Sobolev spaces built upon general Musielak-Orlicz norms is offered. These norms are defined in terms of generalized Young functions which also depend on the x -variable. Under minimal conditions on the latter dependence, a Sobolev conjugate is associated with any function of this type. Such a conjugate is sharp, in the sense that, for each fixed x , it agrees with the sharp Sobolev conjugate in classical Orlicz spaces. Both Sobolev inequalities in the whole Euclidean space and Sobolev-Poincaré inequalities in domains are established. Compact Sobolev embeddings are also presented. In particular, optimal embeddings for standard Orlicz-Sobolev spaces, variable exponent Sobolev spaces, and double-phase Sobolev spaces are recovered and complemented in borderline cases. A key tool, of independent interest, in our approach is a new weak type inequality for Riesz potentials in Musielak-Orlicz spaces involving a sharp fractional-order Sobolev conjugate. This is a joint work with Lars Diening.