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Marcinkiewicz–Zygmund-type sampling discretization for Orlicz-type norms

In the talk we are going to discuss the recent progress in the problem of sampling discretization for Orlicz-type norms. Informally speaking, sampling discretization studies how one can replace the integral norms for a class of functions by the evaluation of these functions at a fixed small set of points. On the one hand, such problems are classical. The first results in sampling discretization were obtained in 1937 by Marcinkiewicz for L^p-norms with $p \in (1, +\infty)$ and by Marcinkiewicz–Zygmund for L^1-norm for the class of univariate trigonometric polynomials. On the other hand, the systematic study of sampling discretization has begun only recently.
In more detail, let $C(\Omega)$ be the space of all continuous functions on some compact subset Ω of $\mathbb{R}^n$ equipped with a probability Borel measure μ. Let L be some N-dimensional subspace of $C(\Omega)$, let $p \in [1, +\infty)$, and $\varepsilon \in (0, 1)$. In the classical Marcinkiewicz-type sampling discretization problem, one aims to determine the least possible integer m such that there are points $x_1, \ldots, x_m \in \Omega$, for which

$$(1-\varepsilon)\|f\|_p^p \leq \frac{1}{m}\sum_{j=1}^{m}|f(x_j)|^p \leq (1+\varepsilon)\|f\|_p^p \quad \forall f \in L,$$

where $\|f\|_p^p := \int_\Omega |f(x)|^p \mu(dx)$ and $\|f\|_\infty := \max\{|f(x)|\colon x \in \Omega\}$. Clearly, m can't be less than the dimension N of the subspace L. Thus, we are interested in the conditions under which the number of points m is close to the dimension N.
The talk will address a modification of this problem when the L^p-norm is replaced with an Orlizc-type norm.

The talk is based on a joint work with Sergey Tikhonov.