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### **On a problem of Lions on real interpolation spaces. The quasi-Banach case**

It is an old problem of Jacques-Louis Lions to determine whether a given family of interpolation spaces really depends on its parameters. For the real method and *Banach couples*  $(A_0, A_1)$  this was solved by Janson, Nilsson, Peetre and Zafran. They showed that if  $A_0 \cap A_1$  is not closed in  $A_0 + A_1$ , then all interpolation spaces  $(A_0, A_1)_{\theta, p}$  with  $0 < \theta < 1$  and  $1 \leq p \leq \infty$  are different from each other. In the proof they used a deep theorem of M. Lévy and duality arguments.

In the talk it will be shown that (under a mild condition on the couple) this result remains true for *quasi-Banach* couples and the *extended range*  $0 < p \leq \infty$  of the second parameter. The proof also uses a variant of Lévy's theorem, but the duality arguments are replaced by certain inductive computations with Peetre's  $K$ -functional.

This is joint work with Fernando Cobos (Madrid) and Michael Cwikel (Haifa).