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### **Fractional Orlicz-Sobolev spaces and modulus of continuity**

Homogeneous fractional Orlicz–Sobolev spaces extend classical fractional Sobolev spaces governed by the Gagliardo–Slobodetskii seminorm, which were introduced in late 1950’s and which are defined in terms of a non-integer smoothness parameter. They provide a natural framework for solutions to nonlocal elliptic problems associated with non-polynomial nonlinearities. This is achieved by replacing the power type integrability with an integrability condition expressed in terms of a Young function. Fractional Orlicz–Sobolev spaces were introduced in 2019 by Fernandez-Bonder and Salort, and their functional properties have been intensively investigated ever since, either out of the pure mathematical curiosity, or in connection with one or more of its many applications.

Like for any type of Sobolev spaces, relations to other function spaces constitute a fundamental issue in the theory of fractional Orlicz–Sobolev spaces, as they provide a crucial tool for transferring regularity from the data to solution in related differential equations. Properties of functions in these spaces are governed by the smoothness parameter and the Young function.

The regularity of functions belonging to fractional Orlicz–Sobolev spaces has first been studied in the so-called subcritical regime, in which sharp embeddings into spaces defined in terms of global integrability properties of functions, called rearrangement-invariant spaces, are the main subject of investigation. The situation in the supercritical regime was tackled later. Here, one can study various finer properties of functions such as criteria for continuity, optimal moduli of continuity, or a control of mean oscillation expressed by a membership into spaces of generalized Campanato type.

The aptness of the notion of fractional Orlicz–Sobolev spaces which we adopt is supported by the fact that, unlike in the classical case, setting the smoothness parameter as an integer exactly matches their counterparts for integer-order Orlicz–Sobolev spaces. Interestingly, customary techniques that have proved appropriate for classical fractional Sobolev spaces, such as characterizations of Hölder spaces in terms of Campanato spaces, Littlewood–Paley decompositions and Hardy-type inequalities fail to yield optimal conclusions for fractional Orlicz–Sobolev spaces. This discrepancy forces us to adopt novel approaches.

In the talk we shall give a survey of recent results on fractional Orlicz–Sobolev spaces obtained jointly with Angela Alberico, Andrea Cianchi and Lenka Slavíková, with particular emphasis on the sharp moduli of continuity of functions from a supercritical space.