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Hardy spaces and harmonic weights

Let L be a non-negative self-adjoint operator on a metric measure space (X, d, μ). Denote by T_t the semigroup generated by L. By definition, a function f belongs to the Hardy space H^1_L if the maximal function $\sup_{t>0} |T_t f(x)|$ is integrable. Such a space is one of the main points of interest in harmonic analysis. One classical result for the classical Laplacian $L = -\Delta$ on $\mathbb{R}^n$ states that a function f is in $H^1_{-\Delta}$ if it can be represented as $f(x) = \sum_k \lambda_k a_k(x)$, where $\sum_k |\lambda_k| < \infty$ and a_k are the so-called atoms, i.e. there are balls B_k such that:

$$\operatorname{supp} a_k \subseteq B_k, \quad ||a_k||_\infty \leq |B_k|^{-1}, \quad \int a_k = 0.$$

Of course, for the classical Laplacian $-\Delta$ there is only one (up to a constant) bounded harmonic function, i.e. the constant function. In the talk we will consider a differential operator L together with the harmonic function h associated with it. We shall present results similar to the classical atomic decomposition theorem given above. The atoms will satisfy similar localization, size, and cancellation conditions, but with respect to the associated harmonic function h. Additionally, we shall consider the case when there are more than one (linearly independent) bounded harmonic functions.

The results were obtained in joint projects with Jacek Dziubański, Adam Sikora, and Lixin Yan.