Muckenhoupt Weights Meet Brezis-Seeger-Van Schaftingen-Yung Formulae in Ball Banach Function Spaces

In this talk, we give two new characterizations of Muckenhoupt weights, via establishing a weighted variant of a profound and far-reaching inequality obtained by A. Cohen, W. Dahmen, I. Daubechies, and R. DeVore in 2003. As an application, we establish a representation formula of gradients with sharp parameters in ball Banach function spaces, which extends the famous formula obtained by H. Brezis, A. Seeger, J. Van Schaftingen, and P-L. Yung in 2021 from classical Sobolev spaces to various different Sobolev-type spaces and gives an affirmative answer to the question on page 29 of [Calc. Var. Partial Differential Equations 62 (2023), Paper No. 234]. These results establish the equivalence among the Muckenhoupt weight, the weighted variant of the inequality of Cohen et al., and the weighted upper estimate of the formula of Brezis et al.